Spread of a Rumor The spread of a rumor in a certain school is modeled by the equation

$$P(t) = \frac{300}{1 + 2^{4-t}},$$

where $P(t)$ is the total number of students who have heard the rumor t days after the rumor first started to spread.

(a) Estimate the initial number of students who first heard the rumor.

(b) How fast is the rumor spreading after 4 days?

(c) When will the rumor spread at its maximum rate? What is that rate?

$P(x) = \frac{300}{1+2^{4-x}} = 300(1+2^{4-x})^{-1}$

a) $P(0) = 17$ or 18

b) $P'(x) = -300(1+2^{4-x})^{-2}(2^{4-x}\cdot \ln 2)(-1)$

$P'(x) = \frac{300\cdot 2^{4-x}\cdot \ln 2}{(1+2^{4-x})^2}$ $\quad P'(4) = 51.986$ or 52 ppl/day

c) $P''(x) = \frac{(1+2^{4-x})^2(300\cdot \ln 2\cdot 2^{4-x}\cdot \ln 2\cdot(-1)) - (300\cdot 2^{4-x}\cdot \ln 2)(2(1+2^{4-x})(2^{4-x}\cdot \ln 2\cdot(-1)))}{(1+2^{4-x})^4}$

$300\cdot 2^{4-x}\cdot \ln 2\cdot 2(1+2^{4-x})\,2^{4-x}\cdot \ln 2\cdot(-1) = (1+2^{4-x})^2\,300\cdot \ln 2\cdot 2^{4-x}\cdot \ln 2(-1)$

$2\cdot 2^{4-x} = 1+2^{4-x}$

$2 = \frac{1+2^{4-x}}{2^{4-x}}$ $\qquad 2 = \frac{1}{2^{4-x}}+1$ $\qquad \frac{1}{2} = 2^{4-x}+1$

$2^{4-x}+2^{4-x} = 1+2^{4-x}$

$2^{4-x} = 1$ $\qquad 4-x = \log_2 1 = 0$

$4-x = 0$ $\qquad \boxed{x=4}$ $\qquad P(4) = 52$ ppl/day

4.1

#11-14

4.2

#6,8,19,33,49

4.3

#4,12,13,31,37

4.5

#4,6,21,23,28,29

Review

#3,11,19,30